Permutations With Like Objects

Ex$_1$: a) How many unique arrangements can be made from SUE?

Ans: $3! = 3\times2\times1 = 6$ $\therefore$ 6 unique arrangements.
More specifically {SUE, SEU, USE, UES, ~~SUE~~, EUS, ESU}

b) How many unique arrangements can be made from ANN$_1$?

Sol: $AN N_1$ N_1AN NN_1A
NAN_1 AN_1N N_1NA

Thus 6 orders. The subscript keeps track of the permutations of letters in ANN.

Note$_1$: If the two N's in a permutation trade places, the resulting permutation is the same as the original one. The two can trade places in $_2P_2 = 2! = 2$ ways.

Note$_2$: Since we counted twice as many arrangements as there really are, the number of unique arrangements of ANN is

$$\frac{3!}{2!} = \frac{3\times\cancel{2!}}{\cancel{2!}} = 3.$$

{ANN, NAN, NNA}

Note$_3$: The number of permutations of **n** objects, of which **a** objects are alike, and another **b** objects are alike, and so on, is:

$$\boxed{\frac{n!}{a!\times b!\times c!\times\ldots}}$$

Ex$_2$: How many permutations are there of your first name? ZEKERIJAH

$n = 9$, 2 E's.

$$\therefore \frac{9!}{2!} = \frac{9\times8\times7\times6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}}$$

$= 181440$ permutations.

Ex$_3$: How many permutations exist from

a) MISSISSAUGA

$$\frac{11!}{2!\,4!\,2!} = 415800$$

b) PRESTONPANTHERS

$$\frac{15!}{2!\,2!\,2!\,2!\,2!\,2!} = 2.0432412E10 = 2.0432412\times10^{10} = ?$$

c) {1,1,1,2,3,3,4,4}

$$\frac{8!}{3!\,2!\,2!} = 1680$$

d){4 yellow, 1 blue, 3 red}

$$\frac{8!}{4!\;3!} = \frac{8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}\;3!} = \frac{8\times7\times\cancel{6}\times5}{\cancel{3\times2\times1}} = 8\times7\times5 = 280$$

Ex$_4$: a) How many arrangements exist of 2 Heads and 1 Tail if one coin is tossed 3 times?

number of tosses → $\dfrac{3!}{2!\,1!} = 3$

number of Heads ← number of Tails

b) P(2 Heads and 1 Tail) $= 3\; P(H)\,P(H)\,P(T)$ ← prob. of tossing a tail

number of perm. of 2 Heads and one Tail

$$= 3\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$
$$= 3\left(\tfrac{1}{8}\right)$$
$$= \tfrac{3}{8}$$

Ex$_5$: A card is drawn from a regular deck that is defaced. i.e. Only the 1-10 cards remain.

a) A card is drawn 4 times with replacement. Find P(Exactly 3 hearts turn up).

$$P(\text{Exactly 3 hearts}) = \frac{4!}{3!\,1!}\; P(\text{Heart})\,P(\text{Heart})\,P(\text{Heart})\,P(\text{anything else})$$
$$= 4\left(\tfrac{10}{40}\right)\left(\tfrac{10}{40}\right)\left(\tfrac{10}{40}\right)\left(\tfrac{30}{40}\right)$$
$$= \cancel{4}\left(\cancel{\tfrac{1}{4}}\right)\left(\tfrac{1}{4}\right)\left(\tfrac{1}{4}\right)\left(\tfrac{3}{4}\right) = \tfrac{3}{64}$$

$\tfrac{3}{64}$

b) Repeat part a) without replacement.

$$P(\text{Exactly 3 hearts}) = 4\left(\tfrac{10}{40}\right)\left(\tfrac{9}{39}\right)\left(\tfrac{8}{38}\right)\left(\tfrac{30}{37}\right)$$
$$= \cancel{4}\left(\tfrac{1}{\cancel{4}}\right)\left(\tfrac{3}{13}\right)\left(\tfrac{4}{19}\right)\left(\tfrac{30}{37}\right)$$
$$= \frac{3\times 4\times 30}{13\times 19\times 37}$$
$$= \frac{360}{9139}$$

Ex$_6$: Two cards drawn with replacement from a standard deck and one die is rolled once.

a) P(2 red cards are drawn and an even number is rolled) $= \dfrac{2!}{2!}\times\dfrac{1!}{1!}\; P(\text{Red})\,P(\text{Red})\,P(\text{even})$

$$= \left(\tfrac{26}{52}\right)\left(\tfrac{26}{52}\right)\left(\tfrac{3}{6}\right)$$
$$= \tfrac{1}{2}\times\tfrac{1}{2}\times\tfrac{1}{2}$$
$$= \tfrac{1}{8}$$

b) P(Exactly 1 red card is drawn and an odd number is rolled) $= \dfrac{2!}{1!\,1!}\times\dfrac{1!}{1!}\; P(\text{Red})\,P(\text{Black})\,P(\text{o}$

$$= 2\left(\tfrac{26}{52}\right)\left(\tfrac{26}{52}\right)\left(\tfrac{3}{6}\right)$$
$$= \cancel{2}\times\tfrac{1}{\cancel{2}}\times\tfrac{1}{2}\times\tfrac{1}{2}$$
$$= \tfrac{1}{4}$$

Homework: Pg. 256: #9 and worksheet