## Permutations With Like Objects

 $Ex_1$ : a) How many unique arrangements can be made from SUE?

Hore specifically [SUE, SEU, USE, UES, SEE,

b) How many unique arrangements can be made from ANN<sub>1</sub>?

Sol: ANH, NAN NHA
Thus 6 orders. The subscript keeps track of the permutations of letters in ANN.

Note<sub>1</sub>: If the two  $N'_{s}$  in a permutation trade places, the resulting permutation is the same as the original one. The two can trade places in  $_{2}$   $_{2}$  = 2! = 2 ways.

Note₂: Since we counted ★₩₩ as many arrangements as there really are, the number of unique arrangements of ANN is

{ ANH, HAN, NMA}

Note<sub>3</sub>: The number of permutations of **n** objects, of which **a** objects are alike, and another **b** objects are alike, and so on, is:

 $Ex_2$ :

How many permutations are there of your first name? ZEKERIJAH 9! = 9x8x7 x 6x5 x4 x3x2. 2! = 181440 permutations.

How many permutations exist from Ex<sub>3</sub>:

> a) MISSISSAUGA 11! = 415 800

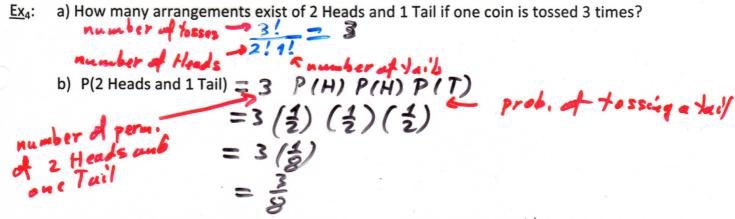
b) PRESTONPANTHERS

15! = 2.04 3 2412 £10

212! 2! 2! 2! 2! = 2.04 3 2412 £10

c) {1,1,1,2,3,3,4,4}

d){4 yellow, 1 blue, 3 red}



Exs: A card is drawn from a regular deck that is defaced. i.e. Only the 1-10 cards remain.

a) A card is drawn 4 times with replacement. Find P(Exactly 3 hearts turn up).  $P(E \times actly 3 \text{ hearts}) = \frac{4!}{3! \, 1!} P(Heart) P(Heart) P(Heart) P(Acart) P(Ac$ 

b) Repeat part a) without replacement.

P (Exactly 3 hearts) = 
$$4 \begin{pmatrix} 39 \\ 40 \end{pmatrix} \begin{pmatrix} 39 \\ 39 \end{pmatrix} \begin{pmatrix} 38 \\ 38 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$$

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 13 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

=  $4 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 34 \end{pmatrix}$ 

Ex<sub>6</sub>: Two cards drawn with replacement from a standard deck and one die is rolled once.

- a) P(2 red cards are drawn and an even number is rolled) =  $\frac{2!}{2!} \times \frac{4!}{2!} P(Red) P(Red) P(even)$   $= \frac{26}{52} \left(\frac{26}{52}\right) \left(\frac{3}{6}\right)$   $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$   $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
- b) P(Exactly 1 red card is drawn and an odd number is rolled) =  $\frac{2!}{4!4!} \times \frac{4!}{4!} P(Red) ?(Black) P(seed) = 2(\frac{26}{52})(\frac{26}{52})(\frac{2}{52}) = 2(\frac{26}{52}) \times \frac{4}{52} \times \frac{4}{52}$

Homework: Pg. 256: #9 and worksheet