Factorial Notation

Working with arrangements, we frequently meet products such as

5x4x3x2x1

9x8x7x6x5x4x3x2x1, and etc.

For a natural number n,

$$\boxed{n! = n\times(n-1)\times(n-2)\times(n-3)\times\cdots\times3\times2\times1}$$

This is read as n factorial.

Note that $0! = 1$ and $1! = 1$.

Ex 1: Calculate each of the following:

a) $4! = 4\times3\times2\times1$
$= 24$
$4! = 4\times(4-1)\times(4-2)(4-3)$

b) $9! = 9\times8\times7\times6\times5\times4\times3\times2\times1$
$= 362\ 880$

c) $1! = 1$

Ex 2: Evaluate each expression.

a) $\frac{8!}{6!} = \frac{8\times7\times\cancel{6!}}{\cancel{6!}}$
$= 8\times7$
$= 56$

b) $\frac{75!}{71!} = \frac{75\times74\times73\times72\times\cancel{71!}}{\cancel{71!}}$
$= 75\times74\times73\times72$
$= 29\ 170\ 800$

c) $\frac{17!}{15!2!} = \frac{17\times16\times\cancel{15!}}{\cancel{15!}\times2\times1}$
$= \frac{17\times\cancel{16}^{8}}{\cancel{2}}$
$= 17\times8$
$= 136$

d) $\frac{9!}{5!\cancel{4!}}\times\frac{5\times\cancel{4!}}{3!2!} = \frac{9\times\overset{4}{\cancel{8}}\times7\times\cancel{6}\times\cancel{5!}\times5}{\cancel{5!}\times\cancel{3}\times\cancel{2}\times1\times\cancel{2}\times1}$
$= 9\times4\times7\times5$
$= 1260$

Ex 3: Simplify each expression.

a) $(n+2)(n+1)! = (n+2)!$

b) $\frac{(n+1)!}{(n-1)!} = \frac{(n+1)\,n\,\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$(n+1-1)$ $(n+1-2)$

$= (n+1)\,n$
$= n^2+n$

Ex 4: How many arrangements of the letters of the word JUSTIN are there?

There 6 letters, so $6! = 6\times5\times4\times3\times2\times1$
$= 720$
$\therefore$ There are 720 arrangements.

Ex 5: Solve for n. $\frac{(n+1)!}{n!} = 9$

$(n+1)! = (n+1)n!$

$\frac{(n+1)\cancel{n!}}{\cancel{n!}} = 9$
$n+1 = 9$
$n = 9-1$
$\therefore \boxed{n = 8}$