Probability with Independent Events

Independent Events: Two events A and B are independent if the occurrence of event A does not change or effect the probability of event B. Thus

When 2 events, A and B are independent, the probability of both events occurring is given by the formula:

$$P(A \cap B) = P(A)\,P(B)$$

This **product rule for independent events** applies to 3 independent sets as well.

Ex. 1: What is the probability of throwing a sum of sevens two times in a row with a pair of dice?

Ans: ∴ P(sum of seven ∩ sum of seven) = P(sum of seven) × P(sum of seven)

$$= \frac{6}{36} \times \frac{6}{36}$$
$$= \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

Ex. 2: A card is drawn and a die is rolled. Find the probability that a diamond is drawn and an even number is rolled.

Ans: ∴ P(diamond ∩ even number) = P(diamond) × P(even number)

$$= \frac{13}{52} \times \frac{3}{6}$$
$$= \frac{1}{4} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

Ex. 3 A card is drawn from a deck of cards. It is replaced, the deck is re-shuffled, and a second card is drawn. What is the probability you will draw a face card on both draws.

Ans: P(face card ∩ face card) = P(face card) × P(face card)

$$= \frac{12}{52} \times \frac{12}{52}$$
$$= \frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$$

B - defect with AC

Ex. 4: A certain car has a 4% chance of having a defect with the transmission, and a 0.45% chance of having a defect with the air conditioning. Find the probability that:

A - defect with transmission

a) P(the car has both defects)

$= P(A \cap B)$

$= P(A) \times P(B)$

$= 0.04 \times 0.0045$

$= 0.00018$

$= 0.018\%$

b) P(the transmission is defective and the air conditioner is not)

$= P(A \cap B')$

$= P(A) \times P(B')$

$= 0.04\,[1 - P(B)]$

$= 0.04 \times (1 - 0.0045)$

$= 0.03982$

$\doteq 4\%$

$P(A') = 1 - P(A)$

c) P(neither the transmission or the air conditioner is defective) $= P(A' \cap B')$

$= P(A') \times P(B')$

$= [1 - P(A)] \times [1 - P(B)]$

$= (1 - 0.04) \times (1 - 0.0045)$

$= 0.95568$

$\doteq 95.6\%$

d) P(at least one of these things is defective) $= P(A \vee B)$

$= P(A) + P(B) - P(A \wedge B)$

$= 0.04 + 0.0045 - 0.00018$

$= 0.04432$

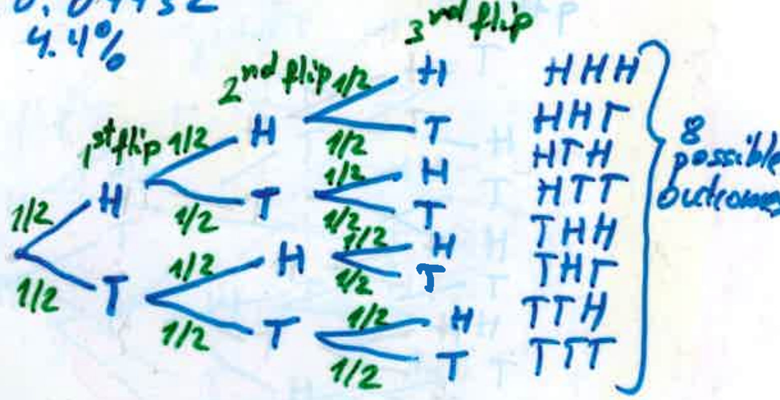
$\doteq 4.4\%$

Ex. 5: Suppose you flip a coin three times. Find

a) P(3^{rd} flip= heads)

b) P(2 heads and 1 tail)

c) P(3 heads in a row)



Ans:

a) P(3^{rd} flip = heads) $= \frac{1}{2}$
$= 50\%$

b) $P(\text{2 heads and 1 tail}) = P(HHT, HTH, THH)$

$= P(HHT) + P(HTH) + P(THH)$

$= P(H)\,P(H)\,P(T) + P(H)\,P(T)\,P(H) + P(T)\,P(H)\,P(H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= \frac{1+1+1}{8}$

$= \frac{3}{8}$

c) $P(\text{3 heads in a row}) = P(HHH)$

$= P(H)\,P(H)\,P(H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

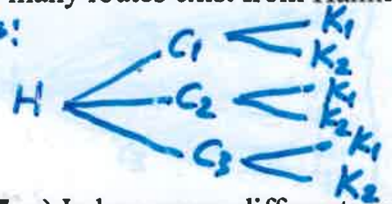
$= \frac{1}{8}$

Note$_1$: The Fundamental Counting Principal

The total number of outcomes is the product of the possible outcomes at each step in the sequence. i.e. if a first action can be performed in 'a' ways, and the second in 'b' ways, and the third in 'c' ways, then these actions can be performed together in $a \times b \times c$ ways.

Ex. 6: A map shows 3 roads from Hamilton to Cambridge and 2 roads from Cambridge to Kitchener. How many routes exist from Hamilton to Kitchener?

Ans:



$3 \times 2 = 6$

$\therefore$ Six routes exist from Hamilton to Kitchener.

Ex. 7: a) In how many different ways can you arrange the word HEXAGON? _ _ _ _ _ _ _

7 6 5 4 3 2 1

$\therefore 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$\therefore$ You can arrange the letters of the word HEXAGON in 5040 different ways.

b) The arrangement must start with an E. E _ _ _ _ _ _

1 6 5 4 3 2 1

$1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$\therefore$ 720 different ways.

Ex. 8: A die is rolled and an equal probability spinner is spun. Find

a) P(a six is rolled $\cap$ B is spun) = P(a six is rolled) $\times$ P(B is spun)

$= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

b) P(even number is rolled $\cup$ C is spun) = P(even number is rolled) + P(C is spun) − P(even number is rolled $\cap$ C is spun)

$= \frac{3}{6} + \frac{1}{6} - \left(\frac{3}{6} \times \frac{1}{6}\right)$

$= \frac{3}{6} + \frac{1}{6} - \frac{1}{12}$

$= \frac{6}{12} + \frac{2}{12} - \frac{1}{12}$

$= \frac{6+2-1}{12}$

$= \frac{7}{12}$

Ex. 9: Find P(Five dice all show the same face).

$= P(1,1,1,1,1) + P(2,2,2,2,2) + P(3,3,3,3,3) + P(4,4,4,4,4) + P(5,5,5,5,5) + P(6,6,6,6,6)$

$= P(1)P(1)P(1)P(1)P(1) + P(2)P(2)P(2)P(2)P(2) + \ldots + P(6)P(6)P(6)P(6)P(6)$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + \ldots + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{6^5} + \frac{1}{6^5} + \frac{1}{6^5} + \frac{1}{6^5} + \frac{1}{6^5} + \frac{1}{6^5}$

$= \frac{6}{6^5} = \frac{1}{6^4} = \frac{1}{1296}$

Ex. 10: Two dice are rolled. Find P(Sum of 10).

Assigned Work: Pg. 245: #1-6