Z-Scores: Finding the mean and the standard deviation

Ex 1: Given $X \sim N(15, \sigma^2)$, $P(X > 17) = 0.41$, find the standard deviation, σ.

Solution:

$P(x > 17) = 0.41$

$1 - P(x < 17) = 0.41$

$1 - 0.41 = P(x < 17)$

$P(x < 17) = 0.59$

$P\left(z < \frac{17-15}{\sigma}\right) = 0.59$

$P\left(z < \frac{2}{\sigma}\right) = 0.59$

$\frac{2}{\sigma} \approx 0.22$ (from z-score table)

$\sigma = \frac{2}{0.22}$

$\sigma \doteq 9.1$

∴ standard deviation is appr. 9.1

Ex 2: A snake farm advertizes that 25% of their snakes are longer than 1.5 m and 10% of them are longer than 2 m. Snake lengths are distributed normally. What is the mean length of the snakes?

Solution: Let X rep. the length of the snakes. $X \sim N(\bar{x}, \sigma^2)$

$P(X > 1.5) = 0.25 \ldots (1)$

$1 - P(x < 1.5) = 0.25$

$1 - 0.25 = P(x < 1.5)$

$P(X < 1.5) = 0.75$

$P\left(z < \frac{1.5 - \bar{x}}{\sigma}\right) = 0.75$

$\frac{1.5 - \bar{x}}{\sigma} = 0.67$ (from the z-score table)

$1.5 - \bar{x} = 0.67\sigma$

$1.5 = \bar{x} + 0.67\sigma$

$\bar{x} + 0.67\sigma = 1.5 \ldots (3)$

$P(X > 2) = 0.10 \ldots (2)$

$P(X < 2) = 0.90$

$P\left(z < \frac{2 - \bar{x}}{\sigma}\right) = 0.90$

$\frac{2 - \bar{x}}{\sigma} = 1.28$ (from the z-score table)

$2 - \bar{x} = 1.28\sigma$

$2 = \bar{x} + 1.28\sigma$

$\bar{x} + 1.28\sigma = 2 \ldots (4)$

Solve the following system of equations by substitution or elimination.

$\begin{cases} \bar{x} + 0.67\sigma = 1.5 \ldots (3) \\ \bar{x} + 1.28\sigma = 2 \ldots (4) \end{cases}$

Multiply (4) by −1

(3) + (4)!

$-0.61\sigma = -0.5$

$\frac{-0.61\sigma}{-0.61} = \frac{-0.5}{-0.61}$

$\sigma \doteq 0.81967$

- Sub. $\sigma \doteq 0.81967$ into (3) or (4) to find $\bar{x}$.

$\bar{x} + 0.67(0.81967) = 1.5$

$\bar{x} + 0.5491789 = 1.5$

$\bar{x} = 1.5 - 0.5491789$

$\bar{x} \doteq 0.9508$

∴ Snakes have a mean length of 0.95 m and a standard deviation of 0.82 m.