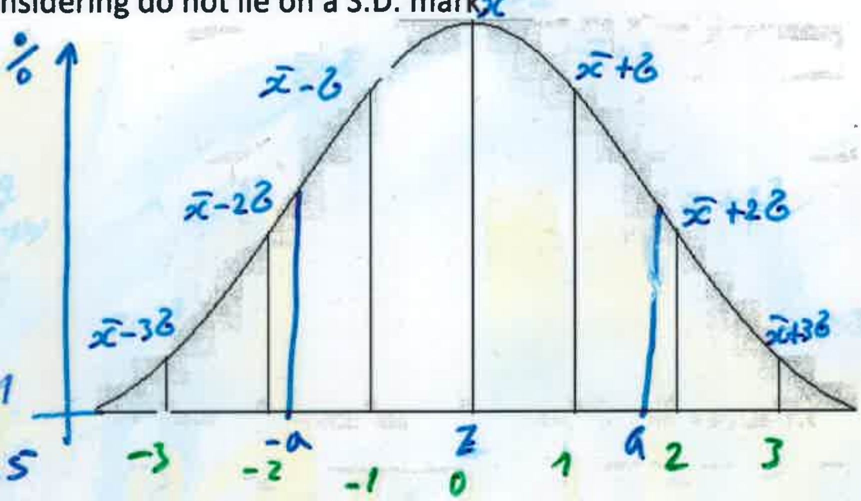
Z-Scores Intervals

Goal: To learn how to use Z-score intervals and the Z-score table.

Recall: We use z-scores to determine the percentage of the population that lies above or below a given value when the x-value(s) we are considering do not lie on a S.D. mark.

"percentage of the data that have a z-score ranging from −3 to +3"

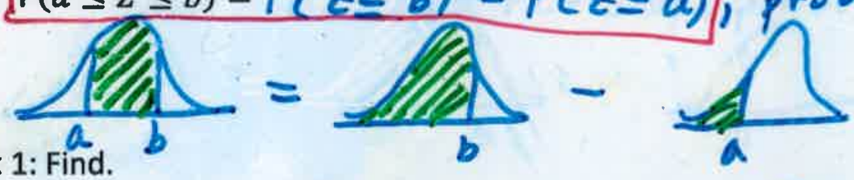


Note the following:

1. $P(-3 \leq Z \leq 3) = P(-3 < Z < 3) = 1$
2. $P(-3 \leq Z \leq 0) = P(0 \leq Z \leq 3) \doteq 0.5$
3. $P(Z \geq a) + P(Z \leq a) = 1$
4. $P(Z \geq -a) = P(Z \leq a) = 1 - P(Z \geq a)$

5. $\boxed{P(Z \geq a) = 1 - P(Z \leq a)} = 1 - P(0 \leq Z \leq a)$

6. $\boxed{P(a \leq Z \leq b) = P(Z \leq b) - P(Z \leq a)}$, provided $a < b$.



Ex 1: Find.

a) $P(X \leq 63)$, *if* $X \sim N(65, 4^2)$.

$\bar{x} = 65$, $x = 63$

$\sigma = 4$

① Find the z-score

$Z = \frac{x - \bar{x}}{\sigma}$

$= \frac{63 - 65}{4}$

$= -0.5$

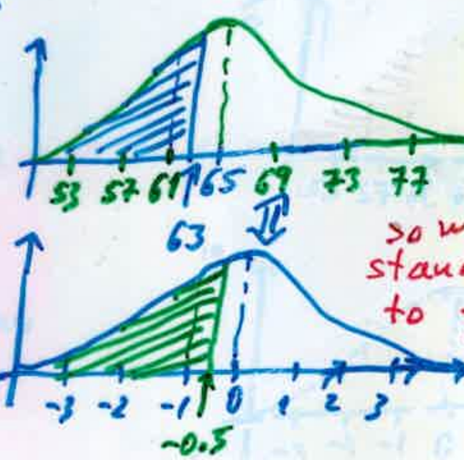
② $P(X \leq 63)$

$= P(Z \leq -0.5)$

$= 0.3085$

$\doteq 30.85\%$

$\therefore$ Approx. 30.85% of the data is less than or equal to 63.



so we have standardized 63 to −0.5 in the $N(0, 1^2)$

b) $P(X \geq 1.5)$, *if* $X \sim N(0, 1)$.

$\bar{x} = 0$

$\sigma = 1$

$P(X \geq 1.5) = 1 - P(X \leq 1.5)$

$= 1 - P(Z \leq \frac{1.5 - 0}{1})$

$= 1 - P(Z \leq 1.5)$

$= 1 - 0.9332$

$= 0.0668$

$\doteq 6.68\%$

$\therefore$ Approx. 6.68% of the data is greater than or equal to 1.5.

Since no distribution is given, then use the standard normal distribution, $N(0, 1^2)$. ($\bar{x} = 0$, $\sigma = 1$)

Ex 2: Calculate

a) $P(-1 < X < 1.5) = P(x < 1.5) - P(x < -1)$

$= P(z < 1.5) - P(z < -1)$

$= 0.9332 - 0.1587$

$\doteq 0.7745$

$\doteq 77.45\%$

$\therefore$ Approx. 77.45% of the data is between -1 and 1.5.

b) $P(70 < X < 80) if X \sim (77.2, 16.0^2)$. $\bar{x} = 77.2, \sigma = 16$, $z = \frac{x - \bar{x}}{\sigma}$

$= P(X < 80) - P(X < 70)$

$= P\left(z < \frac{80 - 77.2}{16}\right) - P\left(z < \frac{70 - 77.2}{16}\right)$

$= P(z < 0.18) - P(z < -0.45)$

$= 0.5714 - 0.3264$

$= 0.245$

$\doteq 24.5\%$

$\therefore$ Approx. 24.5% of the data is greater than 70 and less than 80.

Ex 3: Perch in lake have a mean of 20 cm and a standard deviation of 5 cm. Find the percent of the population that is less than the following lengths (the percentile):

a) 28 cm

$X \sim N(20, 5^2)$

$\bar{x} = 20$

$\sigma = 5$

$P(x < 28)$

$= P\left(z < \frac{28 - 20}{5}\right)$

$= P\left(z < \frac{8}{5}\right)$

$= P(z < 1.6)$

$= 0.9452$

$\doteq 94.52\%$

So, 28 cm is in the 95th percentile interval. Note ~~the~~ $P_{94.52}$ does not exist.

$P_{94} \xleftrightarrow{1\%} P_{95}$

b) 4 cm

$P(x < 4)$

$= P\left(z < \frac{4 - 20}{5}\right)$

$= P\left(z < -\frac{16}{5}\right)$

$= P(z < -3.2)$

$= 0$

So 4 cm is an outlier.

Homework: Pg. 187: #9-13,17,18.