Rational Exponents

Definition of $a^{\frac{1}{n}}$

$a^{\frac{1}{n}}$ is the nth root of a. That is, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

n is a natural number.

$a \geq 0$ if n is even.

Definition of $a^{\frac{m}{n}}$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

m is an integer.

n is a natural number.

$a \geq 0$ if n is even.

Example 1: Evaluate withoug using a calculator.

a) $64^{\frac{2}{3}} = 64^{\frac{2}{3}}$

$= (\sqrt[3]{64})^2$

$= (4)^2$

$= 4 \times 4$

$= 16$

$\sqrt[3]{64} = 4$

$(4 \times 4 \times 4 = 64)$

b) $(-64)^{\frac{1}{3}} = (-64)^{\frac{1}{3}}$

$= \sqrt[3]{-64}$

$= -4$

$(-4)(-4)(-4) = -64$

c) $32^{\frac{4}{5}} = 32^{\frac{4}{5}}$

$= (\sqrt[5]{32})^4$

$= (2)^4$

$= 2 \times 2 \times 2 \times 2$

$= 16$

$\sqrt[5]{32} = 2$

$(2 \times 2 \times 2 \times 2 \times 2 = 32)$

d) $0.04^{\frac{3}{2}} = 0.04^{\frac{3}{2}}$

$= (\sqrt{0.04})^3$

$= (0.2)^3$

$= 0.2 \times 0.2 \times 0.2$

$= 0.008$

Rational exponents are useful for solving equations involving powers.

Example 2: Solve for x. Assume x is positive.

a) $x^2 = 81$

$x^2 = 81$

$\sqrt{x^2} = \pm\sqrt{81}$ ← raise both sides to the exponent of $\frac{1}{2}$ (take the square root of both sides)

$x = \pm 9$

So, $x = -9$ and $x = -9$.

b) $x^3 = 64$

$x^3 = 64$

$(x^3)^{\frac{1}{3}} = (64)^{\frac{1}{3}}$ ← raise both sides to the exponent of $\frac{1}{3}$

$x^{1\times\frac{3}{3}} = \sqrt[3]{64}$ OR $\sqrt[3]{x^3} = \sqrt[3]{64}$

$\therefore \boxed{x = 4}$

c) $x^4 = 81$

$x^4 = 81$

$(x^4)^{\frac{1}{4}} = 81^{\frac{1}{4}}$ ← raise both sides to the exponent of $\frac{1}{4}$

$x^{1\times\frac{4}{4}} = \sqrt[4]{81}$

$x^1 = 3$

$\therefore \boxed{x = 3}$

$\sqrt[4]{81} = 3$ $(3\times3\times3\times3 = 81)$

d) $x^{\frac{2}{3}} = 9$

$x^{\frac{2}{3}} = 9$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = (9)^{\frac{3}{2}}$ ← raise both sides to the exponent of $\frac{3}{2}$

$x^{\frac{2}{3}\times\frac{3}{2}} = (\sqrt{9})^3$

$x^1 = (3)^3$

$x = 3\times3\times3$

$\therefore \boxed{x = 27}$

e) $x^{\frac{3}{2}} = 125$

$x^{\frac{3}{2}} = 125$

$(x^{\frac{3}{2}})^{\frac{2}{3}} = (125)^{\frac{2}{3}}$ ← raise both sides to the exponent of $\frac{2}{3}$

$x^{\frac{3}{2}\times\frac{2}{3}} = (\sqrt[3]{125})^2$

$x^1 = (5)^2$

$\therefore \boxed{x = 25}$

$\sqrt[3]{125} = (125)^{\frac{1}{3}} = 5$ $(5\times5\times5 = 125)$

Homework: Pg. 376: #1-6 odd, 7, 9, 12-14