Fractional Exponents

An exponent that can be written as a fraction of integers is a rational exponent.

In the power law for exponents, $(x^m)^n$, substituting $m=\frac{1}{n}$ gives

$$(x^m)^n = (x^{\frac{1}{n}})^n = x^{(\frac{1}{\cancel{n}})\times\cancel{n}} = x^1 = x$$

So, $(x^m)^n = x$ if $m=\frac{1}{n}$.

If $m=\frac{1}{n}$, then take the n-th root of both side.

$$(x^{\frac{1}{n}})^n = x$$

$$\cancel{\sqrt[n]{}}\,\sqrt{(x^{\frac{1}{n}})^{\cancel{n}}} = \sqrt[n]{x}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$\boxed{\sqrt{x} = x^{\frac{1}{2}}}$

So, $\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$, provided m and n are nutaral numbers.

Example 1: Evaluate the following

a) $25^{\frac{1}{2}}$
$=\sqrt{25}$
$=5$
$(5\times5=25)$

b) $8^{\frac{1}{3}} = 8^{\frac{1}{3}}$
$=\sqrt[3]{8}$
$=2$
$(2\times2\times2=8)$

c) $32^{\frac{1}{5}} = 32^{\frac{1}{5}}$
$=\sqrt[5]{32}$
$=2$
$(2\times2\times2\times2\times2=32)$

d) $-9^{\frac{1}{2}}$
$=(-9)^{\frac{1}{2}}$
$=\sqrt{-9}$
$=\sqrt{(-1)\times9}$
$=\sqrt{-1}\times\sqrt{9}$
$=\underline{i3}$
$i=\sqrt{-1}$
imaginary number

e) $(-27)^{\frac{1}{3}}$
$=(-27)^{\frac{1}{3}}$
$=\sqrt[3]{-27}$
$=-3$
$(-3)(-3)(-3)=-27$

f) $(-8)^{-\frac{1}{3}} = (-8)^{\frac{1}{3}}$
$=\sqrt[3]{-8}$
$=-2$
$(-2)(-2)(-2)=-8$

If n is an even number, then $x \geq 0$. $\boxed{x^{\frac{1}{n}} = \sqrt[n]{x}}$

If n is an odd number, then x can be any real number.

$x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m$ OR $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}$

$= (\sqrt[n]{x})^m$ $\quad = \sqrt[n]{x^m}$

So, $\boxed{x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}}$, where m and n are natural numbers.

Example 2: Evaluate the following:

a) $27^{\frac{2}{3}}$

$= (27)^{\frac{2}{3}}$

$= (\sqrt[3]{27})^2$

$= (3)^2 \leftarrow \sqrt[3]{27} = 3$

$= 3 \times 3$

$= 9$

OR

$\sqrt[3]{27^2} = 9$

b) $100^{\frac{3}{2}}$

$= (100)^{\frac{3}{2}}$

$= (\sqrt{100})^3$

$= (10)^3$

$= 10 \times 10 \times 10$

$= 1000$

OR

$\sqrt[3]{100^2} = 1000$

c) $64^{\frac{2}{3}}$

$= (64)^{\frac{2}{3}}$

$= (\sqrt[3]{64})^2$

$= (4)^2$

$= 4 \times 4$

$= 16$

$\sqrt[3]{64} = 4 \quad (4 \times 4 \times 4 = 64)$

$\sqrt[3]{64^2} = 16$

Homework: Pg. 369: #1-6, 7