Laws of Exponents

Recall:

Positive integer exponent: $x^m = \underbrace{x \times x \times x \times \cdots \times x}_{m \text{ factors}}$

Zero exponent: $x^0 = 1$, provided $x \neq 0$

Negative integer exponent: $x^{-m} = \frac{1}{x^m}$, provided $x \neq 0$

Note: a^{-n} is the reciprocal of a^n

($\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$)

2^{-3} is the reciprocal of 2^3

$\frac{1}{\left(\frac{3}{2}\right)} = 1 \times \frac{2}{3} = \frac{2}{3}$

The exponent laws apply to numerical and variable bases. When the base is a variable, we assume that it is not 0.

Multiplication Law: $x^m \times x^n = x^{m+n}$

Division Law: $\frac{x^m}{x^n} = x^{m-n}$, provided $x \neq 0$

Power of a power law: (m and n are any integer) $(x^m)^n = x^{m \times n}$

Also, $(xyz)^m = x^m y^m z^m$

$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, provided $y \neq 0$.

A simplified algebraic expression contains only positive exponents.

Example 1: Simplify. Evaluate where possible.

a) $b^2 \times b^4 = b^{2+4}$
$= b^6$

b) $b^{-8} \times b^5 = b^{-8+5}$
$= b^{-3}$
$= \frac{1}{b^3}$

c) $(b^2)^3 = b^{2\times 3}$
$= b^6$

d) $b^2 \div b^3 = b^{2-3}$
$= b^{-1}$
$= \frac{1}{b^1}$
$= \frac{1}{b}$

e) $(b^4)^{-1} = b^{4\times(-1)}$
$= b^{-4}$
$= \frac{1}{b^4}$

f) $6^5 \times 6^{-3}$
$= 6^{5+(-3)}$
$= 6^{5-3}$
$= 6^2$
$= 6\times 6$
$= 36$

g) $\frac{(-3)^2}{(-3)^{-1}}$
$=(-3)^{2-(-1)}$
$=(-3)^{2+1}$
$=(-3)^3$
$=(-3)(-3)(-3)$
$=-27$

$\frac{x^2 x^{-6}}{(x^{-2})^3}$

h) $\frac{x^2 x^{-6}}{(x^{-2})^3} = \frac{x^{2+(-6)}}{x^{(-2)\times 3}}$ OR $\frac{\cancel{x^2 x^{-6}}}{\cancel{x^{-6}}}$
$= x^2$

$= \frac{x^{2-6}}{x^{-6}}$
$= \frac{x^{-4}}{x^{-6}}$
$= x^{-4-(-6)}$
$= x^{-4+6}$
$= x^2$

Example 2: Evaluate each expression for a=1, b=-2, and c=3.

a) $(2a^2b^3)^4$

$(2a^2b^3)^4$
$= 2^4 (a^2)^4 (b^3)^4$
$= 16\ a^{2\times4}\ b^{3\times4}$
$= 16\ a^8\ b^{12}$
$= 16\ (1)^8 (-2)^{12} \leftarrow$ sub. a=1 and b=-2
$= 16 \times 1 \times 4096$
$= 65536$

b) $(a^{-3}b)(a^4b^5)$

$(a^{-3}b)(a^4b^5)$
$= a^{-3+4}\ b^{1+5}$
$= a^1\ b^6$
$= (1)\ (-2)^6$
$= 64$

c) $\dfrac{-2ab^5 \times 8a^2b^2}{(4ab^3)^2}$

$\dfrac{2ab^5 \times 8a^2b^2}{(4ab^3)^2} = \dfrac{16\ a^{1+2}\ b^{5+2}}{4^2\ a^2\ b^{3\times2}}$
$= \dfrac{\cancel{16}a^3\ b^7}{\cancel{16}\ a^2\ b^6}$
$= a^{3-2}\ b^{7-6}$
$= a\ b$
$= 1 \times (-2)$
$= -2$

Example 3 (Pg. 363: #8): Computer power has been doubling approximately every 2 years as more and smaller transistors have been integrated to build better computer chips. The number of transistors, T, in a chip has increased according to T = $4500(1.4)^n$, where n is the number of years since 1974. Determine the number of transistors in a computer chip in each year.

a) 1974

Ans:
a) n = 1974 - 1974
= 0
$T = 4500(\cancel{1.4})^{0}$ 1
$= 4500 \times 1$
$= 4500$
∴ 4500 transistors in a computer chip in 1974.

b) 2019

b) n = 2019 - 1974
= 45
$T = 4500\ (1.4)^{45}$
$= 1694236834 0$
∴ 16942368340 transistors in a computer chip in 2019.

$\boxed{\text{Homework: Pg. 362 \#}\underbrace{1 - 6}_{\text{half}}\text{, 10, 11cd, 12, 13, 15}}$