Rearranging Formulas

Formulas usually express one variable in terms of one or more variables. We can use the knowledge of equations and inverse operations to rewrite the formula in terms of a different value.

Example 1: Rearrange the following formulas for the variable indicated.

a) $V = s^3$ for *s*

$\sqrt[3]{V} = \sqrt[3]{s^3}$

$\therefore s = \sqrt[3]{V}$

b) $P = 2w + 2l$ for *w*

$P - 2l = 2w$ ← move 2l to the left side

$2w = P - 2l$

$\frac{2w}{2} = \frac{P-2l}{2}$ ← divide by 2 both sides

$\therefore \boxed{w = \frac{P-2l}{2}}$

c) $\frac{n}{1} = \frac{v}{4l}$ for *l*

$(4l)\,n = (4l)\left(\frac{V}{4l}\right)$ ← multiply both sides by 4l

$4ln = V$

$\frac{4ln}{4n} = \frac{V}{4n}$ ← divide both sides by 4n

$\therefore \boxed{l = \frac{V}{4n}}$

Example 2: Convert 56°C to °F. Use the formula $C = \frac{5(F-32)}{9}$. ...(*)

Ans:

Given: $C = 56°$ celsius

Find: $F = ?$

Method 1: Substitute $C = 56°$ into (*) and solve for F.

$$C = \frac{5(F-32)}{9}$$

$$56 = \frac{5(F-32)}{9}$$

$$9 \times 56 = 1 \times 5 \times (F-32)$$

$$504 = 5(F-32)$$

$$504 = 5F - 160$$

$$504 + 160 = 5F$$

$$664 = 5F$$

$$\frac{664}{5} = \frac{5F}{5}$$

$$132.8 = F$$

So, $F = 132.8°$ Fahrenheit.

Method 2: Rearrange (*) for F, first and then substitute $C = 56$.

$$C = \frac{5(F-32)}{9}$$

$$9C = 5(F-32)$$

$$9C = 5F - 160$$

$$9C + 160 = 5F$$

$$\frac{9C+160}{5} = \frac{5F}{5}$$

$$\boxed{F = \frac{9C+160}{5}} \quad \ldots(**)$$

- Next, substitute $C = 56$ into (**).

$$F = \frac{(9 \times 56) + 160}{5}$$

$$= \frac{504 + 160}{5}$$

$$= \frac{664}{5}$$

$$= 132.8° \text{ Fahrenheit}$$

$\therefore$ $56°$ celsius is equivalent to $132.8°$ farhenheit.

The "isolate, then substitute" and the "substitute, then solve" strategies produce the same result. Sometimes, one strategy is more efficient than the other.

- Isolate the variable first if you will have to calculate it several times. ← Method 2
- Substitute first if the numbers are simple or rearranging the formula is difficult ← Method 1

Example 3: The formula $S = 0.6T + 331.5$ gives the speed of sound in air, S metres per second, at an air temperature of T degrees Celsius. Determine the air temperature for a speed of sound of 336 m/s.

Ans: Given: $S = 336$ m/s, find $T = ?$

Sub. $S = 336$ into $S = 0.6T + 331.5$ and solve for T.

$$336 = 0.6T + 331.5$$
$$336 - 331.5 = 0.6T$$
$$4.5 = 0.6T$$
$$0.6T = 4.5$$
$$\frac{0.6T}{0.6} = \frac{4.5}{0.6}$$
$$\boxed{T = 7.5}$$

∴ The air temperature for a speed of sound of 336 m/s is 7.5° Celsius.

Example 4: In house construction, the safe load, m kilograms, that can be supported by a beam with length l metres, thickness t centimetres, and height h centimetres is given by the formula

$$m = \frac{4th^2}{l} \quad \cdots (1)$$

Determine h when $m = 500$ kg, $l = 4$ m, and $t = 10$ cm.

Ans: Substitute $m = 500$, $l = 4$, and $t = 10$ into (1)

$$m = \frac{4th^2}{l}$$
$$500 = \frac{4 \times 10 h^2}{4} \leftarrow \text{simplify}$$
$$500 = 10h^2$$
$$\frac{500}{10} = \frac{10h^2}{10} \leftarrow \text{divide both sides by 10}$$
$$50 = h^2$$
$$h^2 = 50$$
$$\sqrt{h^2} = \pm\sqrt{50} \leftarrow \text{square root both sides}$$
$$h \doteq \pm 7.1$$

So, $h = -7.1$ is inadmissible. Therefore, $h = 7.1$ cm.

Homework: Pg. 355: #10, 11,14, 15, 16, 18