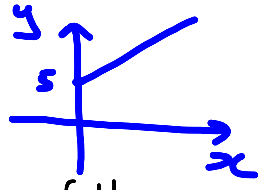
Linear Models

Tables, graphs, and equations are examples of mathematical models.

Mathematical models allow us to represent the relationship between real-world quantities, analyze current behaviour, and predict future behaviour.

You need to be able to identify a linear model in each of the forms.

Table of Values	Graph	Equation
first differences are constant	straight line	y = mx + b



Analyzing the graph of a linear relation

The vertical intercept (b) represents the initial value of the dependent variable.

The slope (m) represents the rate of change in the dependent variable with respect to the independent variable.

Fitting a linear model to data

We can use linear regression to model the data that appear to be linearly related. The closer the regression line is to the data points, the more reliable the predictions are likely to be.

1. Which tables of values model a linear relation? How do you know?

a)

r	0	5	2	3	4	5
C	0.0	31.4	62.8	94.2	125.7	157.1

r	C	First Differences
0	0.0	
		31.4−0=31.4
5	31.4	
		62.8−31.4=31.4
2	62.8	
		94.2−62.8=31.4
3	94.2	
		125.7−94.2=31.5
4	125.7	
		157.1−125.7=31.4
5	157.1	

∴ This table of values models a linear relation since the first differences are equal/constant.

b)

t	0	1	2	3	4	5
h	282.5	272.7	243.3	194.3	125.7	37.5

t	h	First Differences
0	282.5	
		272.7−282.5=−9.8
1	272.7	
		243.3−272.7=−29.4
2	243.3	
		194.3−243.3=−49
3	194.3	
		125.7−194.3=−68.6
4	125.7	
		37.5−125.7=−88.2
5	37.5	

∴ Since the first differences are not the same, then this table of values does not model a linear relation.

2. Which equations model a linear relation? How do you know?

a) $y = -2x$ b) $y = x^2 + 1$ c) $y = 5 - 2x$

a) $y = mx + b$ linear relation

b) $y = ax^2 + bx + c$ quadratic relation

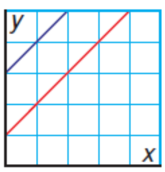
c) $y = mx + b$ linear relation

4. Match each graph with the statement that best describes it.

a) Same initial value, different rates of change

b) Different initial values, same rate of change

c) Different initial values, different rates of change

i)


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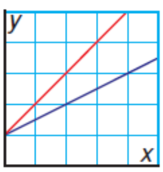
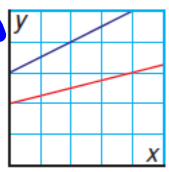
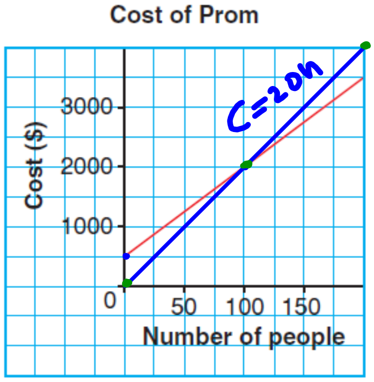


iii)


10. **Assessment Focus** The graduation committee is making arrangements for the prom. This graph shows the total cost of the prom based on the number of people who attend it.



a) What is the vertical intercept?

What does it represent?

- The vertical intercept is $500.
- The vertical intercept represents the initial cost/fee (Even if no one attends, you still pay $500.00)

b) Calculate the rate of change in the cost with respect to the number of people.

What does this rate of change represent?

- (0,500) (100,2000)

$$R.O.Ch = \frac{2000-500}{100-0} = \frac{1500}{100} = 15$$

- The rate of change represents the cost per person ($15 per person)

c) The committee wants to sell tickets at $20 per person. On a copy of the graph, sketch a line that shows the total sales.

$C = 20n$

n	C
0	0
100	2000
200	4000

n – number of people

C – total cost

d) How many tickets would have to be sold to break even?

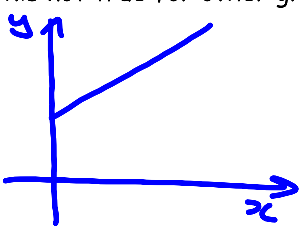
- 100 tickets would have to be sold in order to break even.

e) Suppose the ticket price is $25 per person. How would this change the sales graph? How would this change the break-even point? Justify your answers.

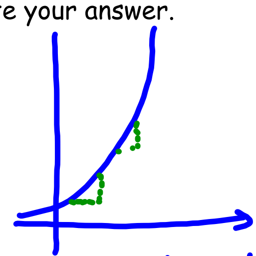
- steeper line
- less people

($\frac{2000}{25} = 80$ tickets need to be sold in order to break even)

Why can we say that rate of change is the same as slope for linear graphs? Why is this not true for other graphs? Use examples to illustrate your answer.



The slope or the rate of change is the same for the line.



Different rate of change.

Homework: Pg. 293: #1,2,4,5,8,10-12,14

#1,2,4,8,10 ← are already done