Optimizing Volume and Surface Area

Among all rectangular prisms with a **given surface area**, a cube has the **maximum volume**.

Among all rectangular prisms with a **given volume**, a cube has the **minimum surface area**.

If there are constraints it may not be possible to form a cube. The maximum volume or minimum surface area occurs when dimensions are closest.

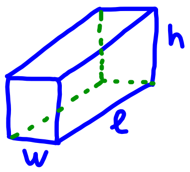
If one or more sides of an object are missing or bordered by a wall or other physical barrier, the optimal rectangular prism is not a cube. You can use diagrams, tables/charts to find the dimensions of the optimal rectangular prism.

A cylinder has optimal values when the height equals diameter.

Recall: The optimal 2D shapes were the circle and the square.

Formulas:

	Rectangular Prism	Cube	Cylinder
Volume	$l \times w \times h$	s^3	$2\pi r^3$
Surface Area	$2(lw+lh+wh)$	$6s^2$	$6\pi r^2$



Sep 14-12:09 AM

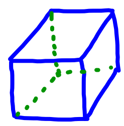
Using optimal dimensions, the formulas for a cylinder would become:

Example 1 (p. 110 #2)

Mathew is constructing a rectangular prism with volume exactly 729 cubic inches. It will have the least possible surface area.

a) Describe the prism. What will be its dimensions?

b) What will be the surface area?



Ans:

a) Aim for a cube.

$V = 729 \text{ in}^3$

$V = s^3 \ldots (1)$

$729 = s^3 \leftarrow$ sub. $V = 729$ in (1) and solve for s

$s^3 = 729$

$\sqrt[3]{s^3} = \sqrt[3]{729}$

$s = (729)^{\frac{1}{3}}$

$\boxed{s = 9 \text{ in}}$

∴ The dimensions of the cube that has the least surface area and has a capacity of 729 in^3 are 9in by 9in by 9in.

b) $S.A. = 6s^2$

$= 6(9)^2 \leftarrow$ sub. $s = 9$

$= 6 \times 81$

$= 486$

∴ The surface area is 486 in^2.

Sep 14-12:13 AM

Example 2 (p. 111 #6)

Jude is designing a plush activity toy for a baby. The toy will be a rectangular prism with surface area of 864 cm^2.

a) Determine the maximum volume of the toy.

b) What are the dimensions of the toy with maximum volume?

Ans: Aim for a cube.

b) $SA = 864\ cm^2$

$SA = 6s^2 \ldots ②$

- Sub. $SA = 864$ into ② and solve for s.

$864 = 6s^2$

$\frac{864}{6} = \frac{\cancel{6}s^2}{\cancel{6}}$

$144 = s^2$

$\sqrt{s^2} = \pm\sqrt{144}$

$\boxed{s = \pm 12}$

So, $s = 12$ cm.

∴ The dimensions of such a cube are 12 cm by 12 cm by 12 cm.

a) Substitute $s = 12$ into $V = s^3$

$V = (12)^3$

$= 12 \times 12 \times 12$

$= 1728\ cm^3$

∴ The maximum volume of the baby toy is 1728 cm^3.

Sep 14-12:15 AM

Example 3 (p. 112 #12)

Filip is designing a can for a new vegetable product. The can should hold 750 mL of vegetables. To reduce waste, he wants the surface area of the can to be as small as possible.

a) What dimensions should Filip use?

b) What will the surface area be?

Ans: Aim for a cylinder, whose height is equal to its diameter.

a) $V = 750$ mL

$= 750\ cm^3$

Recall: $1\ mL = 1\ cm^3$

- Sub. $V = 750$ into $V = 2\pi r^3$ and solve for r.

① $750 = 2\pi r^3$

$\frac{750}{2\pi} = \frac{\cancel{2\pi} r^3}{\cancel{2\pi}}$

$r^3 \doteq 119.4$

$\sqrt[3]{r^3} \doteq \sqrt[3]{119.4}$

$\boxed{r \doteq 4.9\ cm}$

② We know that

$d = 2r$

$d = 2 \times 4.9$

$= 9.8$ cm

∴ The dimensions of the can are 9.8 cm by 9.8 cm.

b) Substitute $r = 4.9$ into $S.A. = 6\pi r^2$

$SA = 6\pi(4.9)^2$

$\doteq 452.6\ cm^2$

∴ The surface area of the can is approx. 452.6 cm^2.

Hwk: Pg. 110: #3-5, 7, 9, 13, 15, 18

Sep 14-12:15 AM