Maximize the Volume of a Cylinder

1. Determine the dimensions of the cylinder with the maximum volume for each surface area. Round the dimensions to the nearest hundredth of a unit.
   a) 1200 cm^2 b) 10 m^2
   c) 125 cm^2 d) 6400 mm^2
2. Determine the volume of each cylinder in question 1. Round to the nearest cubic unit.
3. Many European businesses buy aircraft manufactured in North America. To make the flight home across the Atlantic Ocean, extra fuel tanks are often carried in the cabin of the plane. These extra fuel tanks, called ferry tanks, must be as light as possible. A cylindrical ferry tank is to be made from 8 m^2 of aluminum. What is the maximum volume of fuel that it can hold, to the nearest cubic metre?
4. A fertilizer company wants to make a cylindrical storage container of sheet metal. 72 m^2 of metal is available.
   a) Determine the dimensions of the container with maximum volume. Round the dimensions to the nearest tenth of a metre.
5. **Chapter Problem** Talia ships CDs to her customers in cylindrical plastic containers. The CDs are 12 cm in diameter and 2 mm thick. Talia wants the cylinder to hold as many CDs as possible, but to use as little plastic as possible.
   a) What is the height of the optimal cylinder?
   b) How many CDs will this cylinder hold?
   c) Describe any assumptions you have made.
6. An open-topped cylinder is to be made using 500 cm^2 of plastic.
   a) Describe how you would determine the dimensions of the cylinder of maximum volume.
   b) Determine the dimensions of the cylinder with the optimal volume. Round to the nearest tenth of a centimetre.
7. You have a piece of sheet metal. Your task is to use this material to create a fuel container with maximum volume.
   a) Which shape would have the greatest volume: a square-based prism or a cylinder?
   b) Justify your answer using a fixed surface area of 2400 cm^2.
8. Suppose you have 2000 cm^2 of material to create a three-dimensional figure with the greatest volume. The material can be formed into a square-based prism, a cylinder, or a sphere.
   a) Predict which shape will produce the greatest volume.
   b) Determine the dimensions of each shape so that the volume is maximized.
   c) Determine the volume of each shape.
   d) Was your prediction correct? If not, which of the three shapes has the greatest volume for a given surface area? Will this always be true?
9. **Use Technology** You are to construct a cylinder that has a surface area of 2 m^2. Use a spreadsheet to investigate the dimensions of the cylinder with the greatest volume if
   a) the cylinder has a top and a bottom
   b) the cylinder has no top
10. **Math Contest** Determine the dimensions of the cylinder of maximum volume that can be inscribed in a sphere of radius 8 cm.

Answers

1. a) $h = 15.96$ cm, $r = 7.98$ cm
   b) $h = 1.46$ m, $r = 0.73$ m
   c) $h = 5.16$ cm, $r = 2.58$ cm
   d) $h = 36.86$ mm, $r = 18.43$ mm
2. a) 3193 cm^3 b) 2 m^3
   c) 108 cm^3 d) 39 333 mm^3
3. 2 m^3
4. a) $r = 2.0$ m, $h = 4.0$ m b) 50 265 L
   c) Answers will vary. Example: no metal will be wasted in the building process, no metal is being overlapped
5. a) 12 cm
   b) 60 CDs
   c) Answers will vary. Example: only the dimensions of the CDs need to be considere; no extra space is left for the container's closing mechanism, the plastic container has no thickness.
6. a) Answers will vary. Example: Adjust the surface area formula for the new cylinder, isolate the height and run a few trials using a spreadsheet to find the maximum volume.
   b) $h = 7.3$ cm, $r = 7.3$ cm
7. a) Answers will vary.
   b) cylinder: $r = 11.28$ cm, volume 9018 cm^3; square-based prism: $s = 20$ cm, volume 8000 cm^3
8. a) Answers will vary.
   b) sphere: $r = 12.62$ cm; cylinder: $r = 10.30$ cm, $h = 20.60$ cm, ; square-based prism: $s = 18.26$ cm
   c) sphere 8419.1 cm^3; cylinder 6865.8 cm^3; square-based prism 6088.4 cm^3
   d) The sphere has the greatest volume. Yes, this will always be the case.
   e) For a given surface area: volume of a sphere > volume a cylinder > volume of a square-based prism
9. a) $r = 0.33$ m, $h = 0.63$ m
   b) $r = 0.46$ m, $h = 0.46$ m
10. $r = 6.53$ cm, $h = 9.24$ cm

Minimize the Surface Area of a Cylinder

1. Determine the dimensions of the cylinder with minimum surface area for each volume. Round the dimensions, to the nearest tenth of a unit.
   a) 1200 cm^3 b) 1 m^3
   c) 225 cm^3 d) 4 m^3

2. Determine the surface area of each cylinder in question 1 to the nearest square unit.

3. A cylindrical can is to have a volume of 540 cm^3. What should its dimensions be to minimize the amount of material used to make it? Round the dimensions to the nearest tenth of a centimetre.

4. A cylindrical gas tank is designed to hold 5 L of gas.
   a) Determine the dimensions of the can that requires the least material. Round the dimensions to the nearest tenth of a centimetre.
   b) Describe any assumptions you made in solving this problem.

5. Wade has been asked to design an insulated cylindrical container to transport hot beverages. To keep heat loss to a minimum, the total surface area must be minimized. Find the interior dimensions of the container with volume 12 L that has minimum heat loss. Round to the nearest tenth of a centimetre.

6. A cylindrical can must hold 375 mL of juice.
   a) Determine the dimensions of the can that requires the least amount of aluminum. Round the dimensions to the nearest tenth of a centimetre.
   b) If aluminum costs \$0.001/cm^2, find the cost of the aluminum to make 12 cans.

7. Many of the cans found in our homes are not designed to use the least amount of material. Give reasons why the cans might be designed in other ways.

8. **Chapter Problem** Talia is shipping USB (universal serial bus) cables to a customer. She needs a container with a volume of 500 cm^3 that is as cost efficient as possible. Should she use a square-based prism box or a cylinder for the cables? Justify your answer mathematically.

9. A cylindrical building at Laurentian University in Sudbury, Ontario, is shown in the photo. Do you think it was designed to minimize the amount of heat loss? Justify your answer mathematically.

10. Extra fuel tanks carried in the cabin of a plane are called ferry tanks. These tanks allow a plane to fly greater distances. A cylindrical ferry tank needs to hold 600 L of aircraft fuel.
    a) What are the dimensions of two possible cylindrical fuel tanks?
    b) What should the dimensions of the tank be to minimize the amount of aluminum used in its construction?
    c) How do these dimensions compare to the optimal square-based prism fuel tank?

11. A movie theatre sells popcorn in an open cylindrical container. The large size holds 1500 cm^3 of popcorn.
    a) Determine the dimensions of the container that requires the least amount of cardboard.
    b) How much cardboard is required to make one container?
    c) Describe any assumptions you have made in solving this problem.

12. a) For a given volume, predict which three-dimensional figure will have the minimum surface area: a cube, a cylinder with height equal to diameter, or a sphere.

    b) Check your prediction using the formulas for volume and surface area and a fixed volume of 1000 cm^3.

13. **Math Contest** You are to use 3584 cm^2 of newsprint. Determine the greatest volume that can be completely covered by the newsprint.

14. **Math Contest** Find the dimensions of the square-based prism box with maximum volume that can be enclosed in a cone with base radius 20 cm and height 30 cm.

15. **Math Contest** Find the dimensions that minimize the surface area for a cone with a volume of 225 cm^3.

16. **Math Contest** Find the dimensions of a cone with a surface area of 600 cm^2, if the cone has the greatest possible volume.

Answers:

1. a) $r = 5.8$ cm, $h = 11.6$ cm
   b) $r = 0.5$ m, $h = 1.0$ m
   c) $r = 3.3$ cm, $h = 6.6$ cm
   d) $r = 0.9$ cm, $h = 1.8$ cm
2. a) 634 cm^2 b) 5 m^2
   c) 205 cm^2 d) 15 m^2
3. $r = 4.4$ cm, $h = 8.8$ cm
4. a) $r = 9.3$ cm, $h = 18.6$ cm
   b) Answers will vary. Example: No extra material will be needed to enclose the volume.
5. $r = 12.4$ cm, $h = 24.8$ cm
6. a) $r = 3.9$ cm, $h = 7.8$ cm
   b) \$3.44
7. Answers will vary. Example: It is not always practical to use cylinders with the optimum volume. They may be harder to use, to handle, to carry, or to store.
8. A cylinder will have a surface area of 349 cm^2, and a cube will have a surface area of 378 cm^2. A cylinder is more cost efficient.
9. No, because the cylindrical shape is taller than its diameter. However, there is a large glass area which would encourage solar heating.
11. a) $r = 7.8$ cm, $h = 7.8$ cm
    b) 576 cm^2
    c) Answers will vary. Example: The only cardboard needed is used to enclose the required volume so there is no wastage.
12. a) Answers will vary.
    b) prism 600 cm^2, cylinder 553.7 cm^2, sphere 483.1 cm^2; The sphere has the least surface area.
13. a sphere with volume 20 183 cm^3
14. $s = 26.67$ cm, $h = 10$ cm
15. $r = 4.24$ cm, $h = 11.95$ cm
16. $r = 6.91$ cm, $h = 19.54$ cm