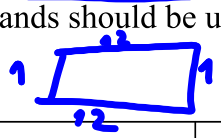
Optimizing Areas and Perimeters (Continued)

There may be restrictions on the rectangle you are optimizing:

- The length and width may have to be whole numbers; or
- The length and width may have to be multiples of a given number.

In these cases, it may not be possible to form a square. The maximum area or minimum perimeter occurs when the length and width are closest in value.

Example 4: Workers at a resort set up a rectangular area to store outdoor equipment and furniture. They use metal stands. They have 26 stands, each 3 m long. The storage enclosure they set up could have different shapes. How many stands should be used for the width and length to make the largest possible enclosure?



Number of stands along the width	Number of stands along the length	Width of enclosure (m)	Length of enclosure (m)	Area enclosed (m2)
1	12	3	36	108
2	11	6	33	198
3	10	9	30	270
4	9	12	27	324
5	8	15	24	360
6	7	18	21	378
7	6	21	18	378
8	5	24	15	360

∴ There should be used seven stands along the width and six stands along the length or vice versa.

OR

$P = 26$

$4s = 26$

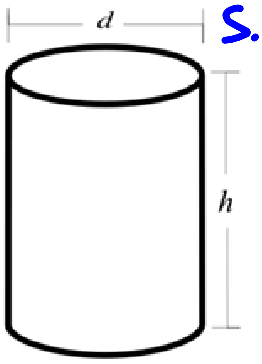
$\frac{\cancel{4}s}{\cancel{4}} = \frac{26}{4}$

$\boxed{s = 6.5}$ ← number of stands

Optimizing Volume and Surface Area of Cylinders

Write a formula for the surface area and the volume of a cylinder with radius, r, and height, h.

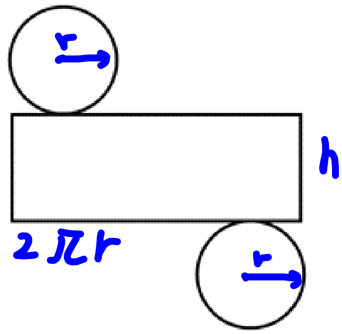
Cylinder



$S.A. = \pi r^2 + \pi r^2 + 2\pi r h$
$= 2\pi r^2 + 2\pi r h$
$= 2\pi r(r + h) \ldots ①$

$V = (\text{base area}) \times \text{height}$
$= \pi r^2 h \ldots ②$

Net of Cylinder



- The minimum surface area for a given volume of a cylinder occurs when its height equals its diameter. That is, h = d or h=2r
- The dimensions of the cylinder of minimum surface area for a given volume can be found by solving the formula V = $2\pi r^2$ for r, and the height will be twice that value, or 2r.
- The maximum volume for a given surface area of a cylinder occurs when its height equals its diameter. That is, h=d or h=2r
- The dimensions of the cylinder with maximum volume for a given surface area can be found by solving the formula $SA=6\pi r^2$, and the height will be twice that value, or 2r.

Sub. $h = 2r$ in ① and ②

$S.A. = 2\pi r(r + h)$
$= 2\pi r(r + 2r)$
$= 2\pi r(3r)$
$= 6\pi r^2$

$V = \pi r^2 h$
$= \pi r^2(2r)$
$= 2\pi r^3$

To be used when optimizing a cylinder

Example 1: a) Determine the least amount of aluminum required to construct a cylindrical can with a capacity of 978 mL. Round to the nearest square centimetre.

Note that 1milliliter = 1cm^3. So, 978 mL = 978 cm^3

① $V = 978\ cm^3$

$V = 2\pi r^3$

$978 = 2\pi r^3$ ← sub. V=978 and solve for r

$\frac{978}{2\pi} = \frac{\cancel{2\pi} r^3}{\cancel{2\pi}}$

$r^3 = \frac{978}{2\pi}$

$r^3 \doteq 155.65$

$\sqrt[3]{r^3} \doteq \sqrt[3]{155.65}$

$\therefore \boxed{r \doteq 5.4\ cm}$

② Calculate the S.A.

- Sub. r = 5.4 into

$S.A. = 6\pi r^2$

$= 6\pi(5.4)^2$

$\doteq 550\ m^2$

∴ The least amount of aluminum required to construct such a can is approx. 550 cm^2.

b) If aluminum costs \$0.001/cm^2, find the cost of the aluminum to make 15 cans.

$S.A. = \underset{\text{number of can}}{15} \times \underset{\text{amount of aluminum required for one can}}{550}$

$= 8250\ cm^2$

Cost = (Cost of aluminum per cm^2) × (Amount required)

$= 0.001 \times 8250$

$= 8.25$

∴ The cost of aluminum to make 15 cans is \$8.25.

Example 2: a) Determine the dimensions of the cylinder with maximum volume that can be made with 950 cm^2 of aluminum.

① $S.A. = 950\ cm^2$

$S.A. = 6\pi r^2$

$950 = 6\pi r^2$ ← sub. S.A.=950 and solve for r

$\frac{950}{6\pi} = \frac{\cancel{6\pi} r^2}{\cancel{6\pi}}$

$r^2 = \frac{950}{6\pi}$

$r^2 \doteq 50.4$

$\sqrt{r^2} = \pm\sqrt{50.4}$

$r \doteq \pm 7.1$

so, $r = 7.1$ cm.

② $d = 2r$

$= 2 \times 7.1$

$= 14.2$ cm

($h = d = 14.2$ cm)

∴ The dimensions of the cylinder are 14.2 cm by 14.2 cm.

b) What is the volume of this cylinder, to the nearest cubic centimetre?

- Sub. $r = 7.1$ into $V = 2\pi r^3$

$V = 2\pi(7.1)^3$

$\doteq 2249\ cm^3$

∴ The volume of this cylinder is approx. 2249 cm^3.

$\boxed{\text{Homework: Pg. 1: \#1 (pick one), 2, 4, 5, 6, 7 \& Pg. 2: \#1 (pick one), 2, 4, 6, 10, try 13}}$