Optimizing Areas and Perimeters

Optimization is the process of finding values that make a given quantity the greatest (or least) possible given certain conditions.

Maximum – greatest possible

Minimum – least possible

Example 1: a) Suppose you have 20 m of fencing. You want to make a rectangular dog pen. What is the maximum area you can provide for a dog?

Ans: $P = 20\ m$

1m, 9m, $A = 9\ m^2$ | 2m, 8m, $A = 16\ m^2$ | 3m, 7m, $A = 21\ m^2$ | 4m, 6m, $A = 24\ m^2$

5m, 5m, $A = 25\ m^2$ ← A square gives us the greatest area

$$\boxed{\begin{aligned} P_{square} &= 4S \\ A_{square} &= S^2 \end{aligned}}$$

S - side length of the square

Algebraically

① $P = 20\ m$

$P = 4s$

$20 = 4s$ ← sub. P=20

$\frac{20}{4} = \frac{\cancel{4}s}{\cancel{4}}$

$\boxed{S = 5}$

② $A = S^2$

$= 5^2$

$= 25$

∴ The maximum area we can provide for the dog is 25 m².

b) Suppose we have a rectangle with an area of 24 units². What dimensions will give us the minimum perimeter?

Ans:

2m, 12m, $P = 28\ m$ | 3m, 8m, $P = 22\ m$ | 4m, 6m, $P = 20\ m$ | 1m, 24m, $P = 50\ m$

Algebraically

① $A = 24\ m^2$

$A = S^2$

$24 = S^2$ ← sub. A=24

$S^2 = 24$ ← solve for s

$\sqrt{S^2} = \pm\sqrt{24}$

$S \doteq \pm 4.9$

$\boxed{S \doteq 4.9\ m}$

② So, the dimensions that provide the minimum perimeter are 4.9 m by 4.9 m.

$$\boxed{\begin{aligned} P &= 4 \times 4.9 \\ &= 19.6\ m \end{aligned}}$$

Conclusion: 1. among all rectangles with a given perimeter, a square has the maximum area.

2. among all rectangles with a given area, a square has the minimum perimeter.

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Example 2: Optimizing with Constraints (restrictions)

A rectangular field bounded on one side by a lake is to be fenced on three sides by 800 m of fence. What dimensions will produce a maximum area?

Ans: $P = 800\ m$

50m, 50m, 700m, $A = 35000\ m^2$ | 100m, 600m, $A = 60000\ m^2$ | 150m, 500m, $A = 75000\ m^2$ | 200m, 400m, $A = 80000\ m^2$

Note that length is twice the width for the figure that provides the maximum area.

w - width

ℓ - length

$\boxed{\ell = 2w}$

Algebraically

① $P = 800\ m$

$P = 4s$

$800 = 4s$

$\frac{800}{4} = \frac{4s}{4}$

$200 = s$

$\boxed{S = 200\ m}$ ← width

② $\ell = 2s$

$= 2 \times 200$

$= 400\ m$

∴ The dimensions that will produce the maximum area are 400 m by 200 m.

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Example 3: You have 400 m of flexible fencing to create an enclosure with the maximum possible area. You could create a rectangular area or a circular area. Which figure encloses the greatest area?

Ans:

① Square

$P = 400\,m$ (P - perimeter)

$P = 4s$

$400 = 4s$

$\frac{400}{4} = \frac{\cancel{4}s}{\cancel{4}}$

$\boxed{s = 100\,m}$

$\boxed{\begin{aligned} A &= s^2 \\ &= 100^2 \\ &= 10000\,m^2 \end{aligned}}$

② Circle

$C = 400\,m$

(C - circumference)

$C = 2\pi r$

$400 = 2\pi r$ ← sub. C=400 and solve for r

$\frac{400}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$

$r = \frac{400}{2\pi}$

$\boxed{r \doteq 63.7\,m}$

$\boxed{\begin{aligned} A &= \pi r^2 \\ &= \pi (63.7)^2 \\ &= 12747.6\,m^2 \end{aligned}}$

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∴ A circle encloses the greatest area. (12747.6 > 10000)

$\boxed{\text{Hwk: Pg. 94: \#4, 10, 12, 14, 17}}$

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