

Applications of Trigonometry

How do we decide whether to use the primary trigonometric ratios (SOH-CAH-TOA), the sine law or the cosine law when solving problems using trigonometry?

Rules:

1. When dealing with right triangles, use the primary trigonometric ratios (sine, cosine, and tangent ratios)
2. If the three sides (SSS) or side-angle-side (SAS) are known, then use the cosine law.
3. Otherwise, use the sine law (2 angles and a side) used when dealing with acute or obtuse triangles

Example 1: Two forest rangers sight a campfire, F, from their observation towers, G and H. Ranger G sights the fire with an angle of depression of 34° . Ranger H sights the fire with an angle of depression of 32° . If the observation towers are 4.0 mi apart, how far is the fire from each observation deck?

Solution: - Draw a diagram

Use the sine law to find g and h.

(one side and two angles given)

$$\angle F = 180^\circ - 34^\circ - 32^\circ = 114^\circ$$

$$\frac{f}{\sin F} = \frac{g}{\sin G} = \frac{h}{\sin H}$$

$$\frac{4}{\sin 114^\circ} = \frac{g}{\sin 34^\circ} = \frac{h}{\sin 32^\circ}$$

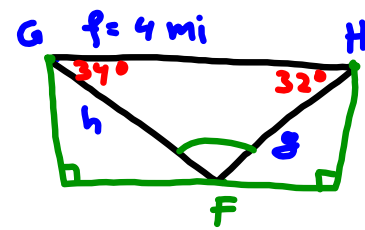
$$\frac{4}{\sin 114^\circ} = \frac{g}{\sin 34^\circ} \quad \leftarrow \text{Finding } g$$

$$\sin 34^\circ \times \left(\frac{4}{\sin 114^\circ} \right) = \cancel{\sin 34^\circ} \times \left(\frac{g}{\cancel{\sin 34^\circ}} \right)$$

$$g = \frac{4 \times \sin 34^\circ}{\sin 114^\circ}$$

$$g = 2.448$$

$$\boxed{g = 2.4 \text{ mi}}$$



Finding h

$$\frac{4}{\sin 114^\circ} = \frac{h}{\sin 32^\circ}$$

$$4.38 = \frac{h}{\sin 32^\circ}$$

$$4.38 \times \cancel{\sin 32^\circ} = h$$

$$4.38 \times \sin 32^\circ = h$$

$$h = 2.327$$

$$\boxed{h = 2.3 \text{ mi}}$$

\therefore The camp fire is approx. 2.3 miles from observation tower G and approx. 2.4 miles from observation tower H.

Example 2: Use this diagram of the rafters in a greenhouse.

a) What angle do the rafters form at the peak of the greenhouse?

b) What angle do they form with the sides of the greenhouse?

Solve this problem using the Cosine Law and using primary trigonometric ratios.

Solution: Let X be the angle the rafters form at the peak of the greenhouse.

a) Use the cosine law to find x (SSS)

$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$

$$\cos X = \frac{17^2 + 17^2 - 26.5^2}{2(17)(17)}$$

$$\cos X = \frac{-124.25}{578}$$

$$\cos X = -0.2150$$

$$\angle X = \cos^{-1}(-0.2150)$$

$$\therefore \boxed{\angle X = 102.4^\circ}$$

\therefore The rafters make an angle of approx. 102.4° .

b) Note that $\angle Y = \angle Z$.

(isosceles triangle) \rightarrow

So, $\angle X + Y + Z = 180^\circ$

$$102.4^\circ + Y + Y = 180^\circ$$

$$102.4^\circ + 2Y = 180^\circ$$

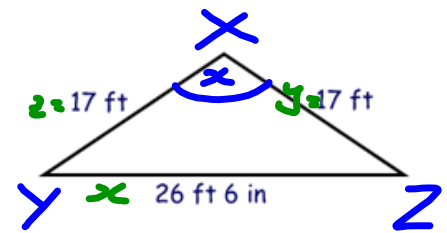
$$2Y = 180^\circ - 102.4^\circ$$

$$2Y = 77.6^\circ$$

$$\frac{2Y}{2} = \frac{77.6^\circ}{2}$$

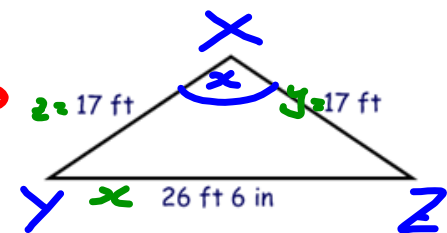
$$\boxed{Y = 38.8^\circ}$$

Homework: Pg. 47: #4, 6, 7, 10



$$1 \text{ ft} = 12 \text{ in}$$

$$26 \text{ ft } 6 \text{ in} = 26.5 \text{ in}$$



\therefore The rafters form an angle of 38.8° with the sides of the greenhouse.