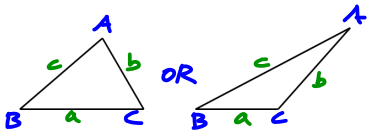
The Cosine Law

Consider the following triangles



The Cosine Law is used in acute and obtuse (oblique) triangles.

$$\begin{cases} a^2 = b^2 + c^2 - 2bc\cos A \\ b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{cases} \quad \text{OR} \quad \begin{cases} a = \pm\sqrt{b^2 + c^2 - 2bc\cos A} \\ b = \pm\sqrt{a^2 + c^2 - 2ac\cos B} \\ c = \pm\sqrt{a^2 + b^2 - 2ab\cos C} \end{cases}$$

Use this version of the cosine law when finding side lengths

- Solve $a^2 = b^2 + c^2 - 2bc\cos A$ for $\cos A$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\frac{2bc\cos A}{2bc} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \angle A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \quad \text{OR} \quad \angle B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \qquad \angle C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

Use this version of the cosine law when finding angles.

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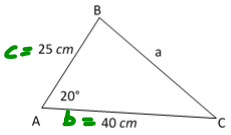
Example 1: Find a.

Ans: Use the cosine law to find a.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 40^2 + 25^2 - 2(40)(25)\cos 20°$$

$$a^2 \doteq 1600 + 625 - 1879.39$$

$$a^2 \doteq 345.61$$

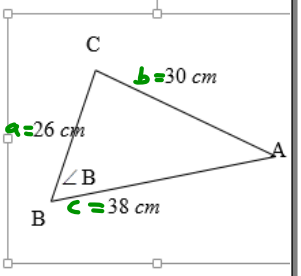
$$\sqrt{a^2} \doteq \pm\sqrt{345.61}$$

$$a \doteq \pm 18.59$$ ← $a = -18.59$ is inadmissible

$$\therefore a = 18.59 \text{ cm}$$

Example 2: Find $\angle B$.

Ans: Use the cosine law to find $\angle B$.



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{26^2 + 38^2 - 30^2}{2(26)(38)}$$ ← substitute $a = 26, b = 30$ and $c = 38$

$$\cos B = \frac{1220}{1976} \quad \text{OR} \quad \cos B \doteq 0.6174$$

$$\angle B = \cos^{-1}\left(\frac{1220}{1976}\right) \qquad \angle B = \cos^{-1}(0.6174)$$

$$\therefore \angle B \doteq 52° \qquad \therefore \angle B = 52°$$

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Example 3: Two boats are approaching an harbour. One boat is approaching on a bearing of 048^0. The second boat is approaching on a bearing of 272^0. What is the measure of the angle between the two boats if measured from the harbour? Draw a diagram to illustrate this problem.

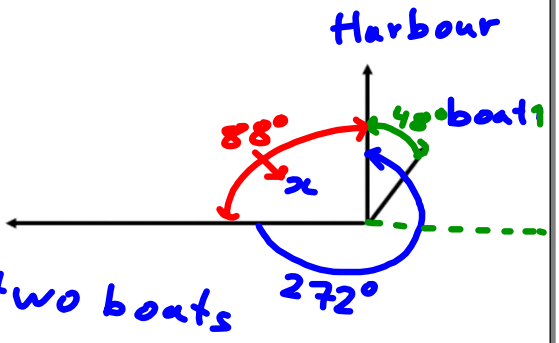
Ans:

$x = 360° - 272°$

$= 88°$

$48° + 88° = 136°$

∴ The angle between the two boats is 136°.



Homework: Pg. 38: #4b, (7 or 8), 9, 10, 11, 13, 16, 17

only one (7a and 8a)

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